Question

A cake is split as follows:

The first person gets 1%

The second person gets 2% of what's left

The third person gets 3% of what's left

...

The last person gets 100% of what's left.

Which person will get the most cake?

No spreadsheets or brute force calculations allowed.

Answer

The 10th person will get the most cake.

Solution

Let f(k) = Cake person k gets.

f(1) = 0.01

f(2) = 0.99 * 0.02

f(3) = 0.99 * 0.98 * 0.03

f(4) = 0.99 * 0.98 * 0.97 * 0.04

And so on...

You can easily see that person 1 will get 1% of the cake and person 2 will get 1.98% . It is obvious each person will get more cake up to a point and then each person after that will get less. The question is where is that maximum share?

For now, let's say there can be a fractional person. Let's find that fraction where the next person will get the same amount. Let's call those two people persons k and k+1.

f(k) = 0.99 * 0.98 * 0.98 * ... * (1 - (k-1)*0.01) * 0.01k

f(k+1) = 0.99 * 0.98 * 0.98 * ... * (1-(k-1)*0.01) * (1 - 0.01k) * 0.01(k+1)

Let's find that value where f(k+1)=f(k).

0.99 * 0.98 * 0.98 * ... * (1-(k-1)*0.01) * (1 - 0.01k) * 0.01(k+1) = 0.99 * 0.98 * 0.98 * ... * (1 - (k-1)*0.01) * 0.01k

Let's divide 0.99 * 0.98 * 0.98 * ... * (1 - (k-1)*0.01) from both sides.

0.01 k = (1 - 0.01k) * 0.01(k+1)

k = (1 - 0.01k) * (k+1)

k = ($\frac{100-k}{100}$) * (k+1)

100k = (100-k)(k+1)

100k = -k^2 + 99k + 100

$k^2 + k - 100 = 0$

Using the quadratic formula…

k = $\frac{1+/-\sqrt{1+400}}{2}$

Only the positive value makes sense so:

k =~ 10.51

This means the pieces increase up to person 10 and then start decreasing with person 11.